Problem 1: Parse Trees (10)

Let  be a grammar with  and these rules:

Give parse trees and derivations for each of the following strings:

1. a + a

A picture containing text, furniture

Description automatically generated

1. a + a ✕ a

Diagram

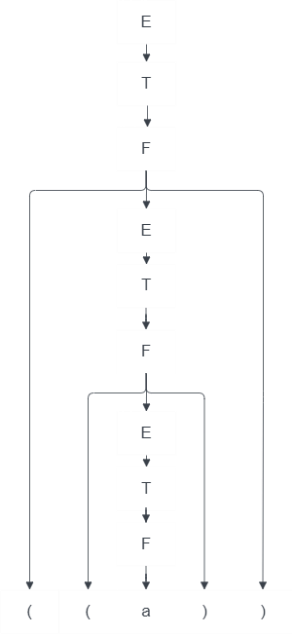
Description automatically generated

1. a ✕ ( a + a )

Graphical user interface, application

Description automatically generated

1. ( ( a ) )



Problem 2: Context Free Grammar (20)

Give context-free grammars generating the following languages, all with :

1. { *s* | *s* starts and ends with the same symbol }

1. { *s* | the length of *s* is odd }

1. {*s* | *s* = *s*R }
2. { a*n*b*n* | *n* ≥ 0 }
3. { *x*1#*x*2#...#*xn* | *n* ≥ 0, each x*i* ∈ Σ\* and for some *i* and *j*, x*i* = x*j*R }

, ,

Problem 3: CFLs versus Regular Languages (15)

Let  be a grammar with , and the set of rules:

1. Describe *L*(*G*) in English.

L(G) is made up of all strings comprised of 0 intersected by at most two # and at least one #. The strings follow the rules where there can only either be one or two #, if there is one # then every time a zero is added on the left of # there will be two added on the right of # (via rule U) or if there are two # there will be any number of zeroes in between, after, or before the two # (via rule T).

1. Prove that *L*(*G*) is not regular.

Assume the language is regular, with . Consider a pumping string . So, we have 𝑠 = 𝑥𝑦𝑧 with 𝑥 the prefix, 𝑦 the cycle string and 𝑧 the suffix, so that:

By definition of , and our pumping string s, all divisions of s must be:

* , where b must be greater than 0
  + Since , the first must be made of x and y, with the rest made of z
* So, the resulting string should be equal to:
* by, set and consider
  + Since which simplified is
  + That means it violates our language, since there is only one # it must follow the rule, which breaks
  + Then by definition of

So, for every possible construction of 𝒚, 𝒙𝒚𝒚𝒛 ∉ 𝑳, →← PL1. By contradiction, the language must be irregular.

Problem 4: Chomsky Normal Form, Easy (5)

Convert these rules to CNF.

Add start and remove :

Remove singe rewrites:

Remove mixed/multiple terminals:

Remove long rewrites to get CNF:

Problem 5: Chomsky Normal Form, Less Easy (15)

Convert these rules to CNF.

Add start and remove :

Remove single rewrites:

Remove mixed/multiple terminals:

Remove long rewrites to get CNF:

Problem 6: Non-Context-Free Languages (20)

Show that these languages are not context-free:

1. (10) The language of palindromes over {0, 1} containing equal numbers of 0's and 1's.

Assume the language is regular, with . Consider a pumping string So, we have , so that:

By definition of , and our pumping string s, all divisions of s must be:

* Since , by PL3 and PL2
  + Either and represent one of the digits
  + Either and represent one digit and represent another
  + Or represents one digit and and represents another
* by, set and consider
  + By definition of palindromes, the leftmost 1 has to be equal to the rightmost 1 and 0 must be an even length
  + In every case would mean that at least one number would not be able to follow the definition of palindromes, thus violates our language
  + Then by definition of

So, for every possible construction of **v** and 𝒚, 𝒛 ∉ 𝑳, →← PL1. By contradiction, the language must be irregular

1. (10) The language of strings over {1, 2, 3, 4} with equal numbers of 1's and 2's **and** equal numbers of 3's and 4's.

Assume the language is regular, with . Consider a pumping string . So we have , so that:

By definition of , and our pumping string s, all divisions of s must be:

* Since , by PL3 and PL2
  + Either , , and represent one symbol
  + Either represents one symbol and and represent another
  + Or , represents one symbol and represents another
* by, set and consider
  + In every case, would cause at least one symbol to become unequal with its counterpart (1 and 2 or 3 and 4)
  + Then by definition of

So, for every possible construction of **v** and 𝒚, 𝒚𝒚𝒛 ∉ 𝑳, →← PL1. By contradiction, the language must be irregular

Problem 7: CFG/PDA Generation (15)

Let Σ={0,1} and *L* be the language of all strings with at least one **1** in their second half.  Give both a CFG that generates *L* and a PDA that recognizes it.

CFG

PDA

